A phase transition model for metric fluctuations in vacuum

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Abstract

Regarding metric fluctuations as generating *roughness* on the fabric of the otherwise smooth vacuum, it is shown that in its simplest form, the effect can be described by the scalar ϕ^4 model. The model exhibits a second order phase transition between a smooth (low-temperature) phase and a rough (high-temperature) one, corroborating the absence of metric fluctuations at low energies. In the rough phase near the critical point, vacuum is characterized by a power-law behavior for the fluctuating field with critical exponent $\beta \approx 0.33$.

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By inducing fluctuations in the metric, quantum effects are generally expected to play a principle role in the structure of spacetime at the Plank scale. Apart from any residual effects they may have on larger scales, these fluctuations become an inseparable part of the spacetime itself when considering its structure down to such fine scales, i.e., at temperatures near as high as the Plank temperature, $T_P \sim 10^{19}\ GeV$. Close to T_P, one may visualize the effect as giving rise to small scale curvature (*roughness*) on the fabric of the otherwise smooth spacetime. Motivated by this picture, we show that in vacuum such metric fluctuations, in simplest form, can be modelled by the scalar ϕ^4 theory. The model exhibits a phase transition at the renormalized Plank temperature $T_P(\infty) > T_P$, the transition being of second order between a smooth phase ($T < T_P(\infty)$) and a rough one ($T > T_P(\infty)$), thus corroborating the absence of metric fluctuations at low energies. In the rough phase near $T_P(\infty)$, vacuum is characterized by a power-law behavior for the fluctuations with critical exponent $\beta \approx 0.33$.

In the following, we shall appropriately phrase the vacuum in presence (absence) of metric fluctuations as the real (ideal) vacuum.

In accordance with general relativity, we take the ideal vacuum to be a de Sitter spacetime with the line element

$$ds^2 = dt^2 - e^{2Ht}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\ d\varphi^2), \quad H = \sqrt{\frac{8\pi G\rho}{3}} \tag{1}$$

where ρ is the energy density of the vacuum. Neglecting fluctuations, it is naively expected that $\rho \sim m_P^4$, m_P being the Plank mass which is of the order of $10^{19}\ GeV$.

Our model Hamiltonian will be defined by the change in vacuum energy due to metric fluctuations, which we now proceed to consider. Since we are not interested in the dynamics of fluctuations we take t=const., thus concentrating on the spatial cross section $dt = 0$ of the vacuum spacetime near the Plank time. Consequently, hereafter, by vacuum this spatial cross section is meant. Noting that the ideal vacuum is Euclidean, we denote its cartesian coordinates by x^i ($i = 1, 2, 3$). Metric fluctuations are to be regarded so as to generate roughness on the fabric of this ideal smooth vacuum. One way to model these fluctuations would, then, be to view their effect extrinsically and represent the real vacuum by a hypersurface in a higher dimensional Eulidean space. In its simplest form, the effect of fluctuations may described by a scalar field $\phi(x^i)$ so that the real vacuum can be represented by the hypersurface $x^4 = \phi(x^i)$ in a four dimensional Euclidean space with cartesian coordinates (x^i, x^4); the hyperplane $x^4 = 0$ (corresponding to $\phi = 0$) representing the ideal

vacuum, of course. The local unit normal to the hypersurface $x^4 = \phi(x^i)$ will then be given by

$$[1 + (\nabla\phi)^2]^{-1/2}(-\nabla\phi, 1) \tag{2}$$

By projecting an element dv of this hypersurface onto the hyperplane $x^4 = 0$, it is readily shown that

$$dv = [1 + (\nabla\phi)^2]^{1/2}\ d^3x \tag{3}$$

This is the volume element of the real vacuum. In absence of fluctuations ($\phi = 0$), (3) reduces to the volume element of the ideal flat vacuum, as it should. Thus, assuming that real vacuum is not too rough ($|\nabla\phi| \ll 1$), the contribution of (3) to the excess vacuum energy due to fluctuations will be

$$\int \frac{1}{2}\ \rho(\nabla\phi)^2\ d^3x \tag{4}$$

The integrand, which arises as a result of volume expansion due to the metric fluctuations, constitutes the "kinetic term" of our model Hamiltonian. The complete Hamiltonian has to take into account the interaction energy of the fluctuations, represented by the "potential term" $V(\phi)$, too. Then, naturally assuming that these fluctuations are scale invariant, their scaling behavior is to be studied via the renormalization group techniques. Indeed, it is this renormalizability requirement that fixes the form of $V(\phi)$ unambiguously. Thus bearing in mind that our model is three dimensional and that $V(\phi)$ should be even in ϕ, we take

$$V(\phi) = A(T)\phi^2 + B(T)\phi^4, \quad B(T) > 0 \tag{5}$$

where T is the temperature. Terms beyond ϕ^6 have been omitted because they represent irrelevant (non-renormalizable) interactions. Also, the ϕ^6 term has been deliberately made irrelevant by taking $B(T) > 0$ and, therefore, ignored. This is because the latter term pertains to tricritical behavior [1, 2] which does not seem to be related to our problem. Now, as is well known, the above potential shows phase transition provided that $A(T)$ vanishes at a certain critical temperature which is naturally to be taken as T_P. This, being the temperature near which metric fluctuations are naively expected to occur, is of course the trial critical temperature; the true critical temparature will be given by the renormalization of this trial value due to the ϕ^4 term. Noting that our model is only applicable in the vicinity of the transition point, we can replace $B(T)$ by its value, b, at T_P and write

$$A(T) = \frac{1}{2}\ a(T - T_P) \tag{6}$$

where the constant a is taken to be negative because we want the high (low) temperature phase to be the rough (smooth) phase.

Thus, the change in vacuum energy due to metric fluctuations that defines our model Hamiltonian is given by

$$H[\phi] = \int d^3x \, \{\frac{1}{2} \, \rho(\nabla\phi)^2 + \frac{1}{2} \, a(T - T_P)\phi^2 + b\phi^4\} \tag{7}$$

where $a < 0$ and $b > 0$. This is the standard ϕ^4 model which, as is well known (see e.g. [3, 4]), exhibits a second order phase transition at the critical temperature $T_P(\infty) > T_P$ given by the zero of the renormalized value of $A(T)$. The high temperature phase ($T > T_P(\infty)$) near the critical point is characterized by the scale-invariant behavior

$$\phi \sim (T - T_P(\infty))^\beta \tag{8}$$

and is therefore rough, while in the low temperature phase ($T < T_P(\infty)$) $\phi = 0$ so that the vacuum is smooth. The critical exponent β is found from the standard renormalization of the ϕ^4 model [3, 4] to have the approximate value 0.33.

Of course, in the above model, we have simply assumed that the fluctuating metric components somehow couple to produce a single fluctuating field $\phi(x)$; we have not considered the possible physical mechanisms (if any) behind the coupling. However, this assumtion, although it simplifies the resulting model considerably, is not fundamental to our approach. What we are essentially pointing out here is that metric fluctuations can be described by phase transition models corroborating the fact that they show no gross effect at 'low' energies.

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